

A new approach for online multiobjective optimization of mechatronical systems

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October 1, 2004

Abstract

We present a new concept for online multiobjective optimization which has been applied to a real-world application, that is the optimization of the assignment of an operating point to a doubly fed linear motor. This problem leads to a time-dependent multiobjective optimization problem. In contrast to classical optimization where the aim is to find the (global) minimum of a single function, we want to minimize simultaneously k objective functions. The solution to this problem is given by the set of optimal compromises, the so-called Pareto set. In the case of the linear motor there are two conflicting aims that have both to be maximized: the degree of efficiency and the converter-utilization factor. The objective functions depend on velocity, force and power, which can be modeled as time-dependent parameters. For a fixed point of time, the entire corresponding Pareto set can be computed by means of a recently developed set-oriented numerical method. An online computation of the time-dependent Pareto sets is not possible, because the computation itself is too complex. Therefore, we combine the computation of the Pareto set with numerical path following techniques. Under certain smoothness assumptions the set of Pareto points can be characterized as the set of zeros of a certain function. Here, path following allows to track the evolution of a given solution point through time.

Key words. multiobjective online optimization, time-dependent Pareto sets, decision making, path following methods, linear motor.

1 Introduction

For the optimization of mechatronical systems one has to bear in mind many things.

- It is possible that conflicting objectives have to be optimized at the same time.
- The objectives can depend on time or other parameters.
- The solutions have to be adjusted online – that is, one has to achieve low computational effort for the optimization.

The purpose of this work was to develop an optimization strategy that allows to account for all these aspects in an adequate way and to apply this strategy to the determination of optimal operating points of a linear motor.

The outline of this contribution is as follows: Section 2 gives an introduction into the field of multiobjective optimization. The main idea in multiobjective optimization is to develop an optimization theory that allows to optimize several conflicting objectives at the same time. For objective functions that additionally depend on time, our idea was to use so-called numerical path following methods which allow to track solutions of parameter-dependent (nonlinear) systems of equations (see Section 3). The following section explains how we combine time-dependent multiobjective problems and numerical path following methods. The resulting optimization algorithm is applied to develop an optimal strategy for the assignment of an operating point to a linear motor in railway vehicles which is described in detail in the last section. This example shows that the mathematical techniques developed in this work are not only of theoretical interest but are also quite useful in mechatronical applications.

2 Multiobjective Optimization

In classical optimization the goal is to find the (global) minimizer of one single objective function $f : \mathbb{R}^n \rightarrow \mathbb{R}$, $n \in \mathbb{N}$. But many problems e.g. in engineering show that it is not clear what "the objective function" is. There often arise several objectives which are conflicting but which have to be optimized at the same time. The application considered in this work – that is the operating point assignment of a linear motor – provides us with two conflicting objective functions. Hence multiobjective optimization techniques are well qualified to solve the problems. Mathematically, an unconstrained *multiobjective optimization problem* is given by

$$\min\{F(x) : x \in \mathbb{R}^n\}, \quad (MOP)$$

where F is defined as the vector of the objective functions $f_1, \dots, f_k : \mathbb{R}^n \rightarrow \mathbb{R}$,

$$F : \mathbb{R}^n \rightarrow \mathbb{R}^k, F(x) = (f_1(x), \dots, f_k(x)).$$

We obviously have to explain, what 'min' means in this context, because we want to minimize a vector-valued function. Therefore we define the following partial order \leq_p on \mathbb{R}^k :

DEFINITION 2.1 Let $u, v \in \mathbb{R}^k$. Then the vector u is *less than* v , if

$$u_i \leq v_i \quad \text{for all } i \in \{1, \dots, k\}.$$

In this case we write $u \leq_p v$.

Using this relation, we can define a solution of (MOP). This solution is not a single optimum, but a set of optimal compromises.

DEFINITION 2.2 A point $x^* \in \mathbb{R}^n$ is called *globally Pareto optimal* for (MOP) (or a *global Pareto point* of (MOP)), if there is no $x \in \mathbb{R}^n$ with

$$F(x) \leq_p F(x^*) \text{ and } f_j(x) < f_j(x^*) \text{ for at least one } j \in \{1, \dots, k\}.$$

If this property is only valid inside a neighborhood $U(x^*) \subset \mathbb{R}^n$, we call x^* *locally Pareto optimal* (or a *local Pareto point*).

From this definition it is not clear how to compute the entire Pareto set efficiently. The following famous theorem of Kuhn and Tucker [7] provides us with a necessary condition for Pareto optimality.

THEOREM 2.3 (KUHN-TUCKER) *Let x^* be a local Pareto point. Then there exist non-negative multipliers $\alpha_1, \dots, \alpha_k \in [0, 1]$ such that*

$$\sum_{i=1}^k \alpha_i \nabla f_i(x^*) = 0 \quad \text{and} \quad \sum_{i=1}^k \alpha_i = 1. \quad (2.1)$$

This condition is obviously not sufficient in general, but it is necessary and sufficient, if the objective functions are convex. Numerical methods for the computation of the Pareto set often use this theorem e.g. for constructing a descent direction [12]. Following [8] we define

DEFINITION 2.4 If $x \in \mathbb{R}^n$ satisfies the Kuhn-Tucker condition (2.1) then it is called a *substationary point*.

Recently a set-oriented numerical method for the computation of the entire Pareto set has been developed (see [2], [13], [14]) and is implemented in the software package GAIO¹. This approach is very useful for our applications, because it comes up with a fine covering of the global Pareto set in a comparatively short computational time.

¹Global Analysis of Invariant Objects, <http://math-www.uni-paderborn.de/~agdellnitz>

When optimizing mechatronical systems it frequently happens that the objective functions additionally depend on time. In the applications we consider (cf. Section 5), time plays an important role. For different points of time the objective functions have different function values, that is we have to optimize

$$f_1(x(t), t), \dots, f_k(x(t), t)$$

simultaneously. The solution of the corresponding multiobjective optimization problem is the set of all possible curves $x(t)$ consisting of Pareto points. Such curves can for example be numerically computed by solving the corresponding multiobjective optimization problems for a discrete covering of the time–interval one wants to consider. But this is computationally costly and in terms of mechatronical systems, where solutions have to be adapted online, not of practical interest.

Secondly, after having computed the entire Pareto set one has to decide which point in this set "matches best" for the underlying system. This process is called "*decision making*". The person who has to determine this point, e.g. the engineer, is called "*decision maker*" (cp. [8]). Due to feasibility reasons the latter instance was substituted by decision heuristics, which which can provide us with good operating points online (see for example [11]).

These two facts show that it is more reasonable to investigate in how to compute a "good path" without the global information about every Pareto set. On the other hand we do not want to lose information about the structure of global Pareto sets completely. In this context, numerical path following methods are a useful tool. These methods allow to track the evolution of a given solution point, which is a zero of a certain function, through time. Numerical path following methods will be described in the next section and in Section 4 we explain, how these methods can be used in terms of online multiobjective optimization.

3 Numerical path following methods

The aim of this work is to develop numerical techniques for the treatment of time–dependent multiobjective optimization problems. This task involves the problem of finding the set of zeros of a smooth parameter–dependent function $H : \mathbb{R}^N \times \mathbb{R} \rightarrow \mathbb{R}^N$ as we will see in Section 4. Very useful techniques for following solutions under the variation of a parameter, which in our special case is the time t , have already been developed (see e.g. [1], [4]). In the following, we shortly outline how these so–called "*numerical path following methods*" work in principle.

Consider a time–dependent nonlinear equation system

$$H(y, t) = 0, \tag{3.2}$$

where $y \in \mathbb{R}^N$ and $t \in \mathbb{R}$. In the context of the algorithms for online multiobjective optimization proposed in this work we will have $y = (x, \alpha)$, where α denotes the weight of the objectives defined by the Kuhn–Tucker system (2.3).

Let $u_0 = (y_0, t_0)$ be a solution of (3.2), i.e. $H(u_0) = 0$. Suppose that the Jacobian $H'(u_0)$ has full rank. In this case the solution set $H^{-1}(0)$ can locally be parametrized by one parameter s in the neighborhood of u_0 . Thus, one obtains a solution curve $c(s)$ with

$$c(0) = u_0 \quad \text{and} \quad H(c(s)) = 0.$$

Differentiating this last equality we obtain

$$H'(c(s))c'(s) = 0.$$

Therefore $c'(s)$ spans the one-dimensional kernel of $H'(c(s))$. By choosing s to be the arclength it follows that $\|c'(s)\| = 1$. Moreover one can show that $c(s)$ is the solution of the initial value problem

$$\begin{aligned} \dot{u} &= T(H'(u)) \\ u(0) &= u_0. \end{aligned} \tag{3.3}$$

Here $T(A)$ denotes for a matrix $A \in \mathbb{R}^{N \times (N+1)}$ with $\text{rank}(A) = N$ the unique vector T satisfying the following conditions:

$$AT = 0, \quad \|T\| = 1 \quad \text{and} \quad \det \begin{pmatrix} A \\ T^T \end{pmatrix} > 0.$$

The last condition allows to fix the orientation of the tangent vector.

A numerical path following method generates a series of points u_i , $i = 0, 1, 2, \dots$ in the following way. Let u_0 an initial point satisfying $H(u_0) = 0$. Then u_{i+1} is generated inductively in two steps (cp. Figure 1).

Predictor Step: Solve (3.3) numerically, e.g. by the explicit Euler method:

$$v_{i+1} = u_i + h \cdot T(H'(u_i)),$$

where $h > 0$ is a certain steplength.

Corrector Step: Here one uses the fact that $c(s)$ solves $H(c(s)) = 0$. Define w_{i+1} to be that point on the curve c , which is closest to v_{i+1} , i.e. one solves

$$\min\{\|v_{i+1} - w\| : H(w) = 0\}.$$

The solution of this problem – this is typically done by Newton’s method – defines the new point u_{i+1} .

4 Online multiobjective optimization

In [3] theoretical concepts for the combination of parameter-dependent multiobjective optimization problems and numerical path following methods are described in detail.

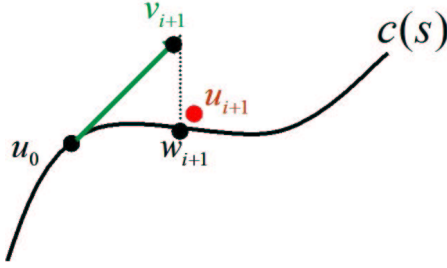


Figure 1: Illustration of the predictor and corrector step in path following methods

For the special case of time-dependent multiobjective optimization problems these techniques are very useful, because computing a path between sets of substationary points can be performed much faster than computing the entire Pareto sets.

In order to combine time-dependent multiobjective optimization problems and numerical path following methods we use the necessary condition given by the Kuhn–Tucker–Theorem. The equation system stated in this theorem consists of $n + 1$ equations and $n + k + 1$ variables ($x \in \mathbb{R}^n$, $\alpha \in \mathbb{R}^k$, $t \in \mathbb{R}$). For a fixed value of the vector α there are n equations and $n + 1$ variables left. For this kind of system, path following methods can be applied directly.

Having computed the entire Pareto set for an initial point in time, e.g. $t = 0$, it is sensible to choose one point in this set as the actually best adjustment of the system and fix the corresponding weight vector α , because the underlying ratio of the objectives can remain the same for the next few instants. The choice of the actually best adjustment can be realized by a certain decision heuristic, which has to be tailored to the special applications under construction (cp. [11]). The decision heuristic for the application considered in this work is described in Section 5. In a second step this decision heuristic can contain a check how flat (assuming an adequate scalarization) the Pareto set in the image space is. It is not recommendable to choose such a Pareto point as the actual best adjustment, where the Pareto set in the image space is flat, because only little losses in one objective can cause great benefits to the other. This fact is visualized schematically in Figure 2.

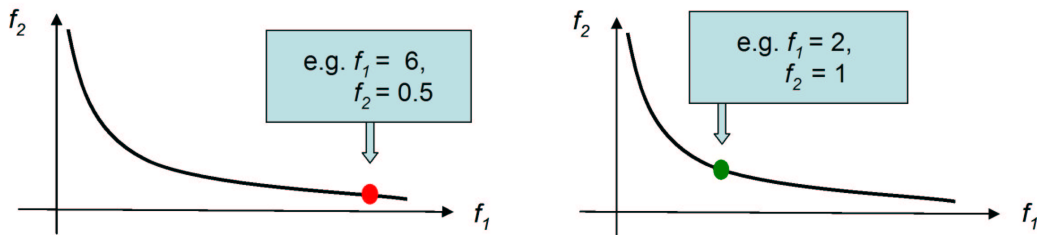


Figure 2: Schematic visualization of a possible second step in a decision heuristic

In order to update the optimal solutions online, we suggest the following procedure:

- I. Compute the entire Pareto set for the given problem at an initial point in time.

Based on a certain decision heuristic the actual best adjustment for the system is calculated.

- II. The corresponding ratio of the objectives is set fixed for a moment. Using path-following techniques, we compute points on the time-dependent Pareto sets.
- III. After a certain time, the entire actual Pareto set is recomputed, the decision heuristic is applied again, and the adjustment of the system is updated. Then we proceed with step II.

There are different ways how to decide that the new entire Pareto set has to be computed in step III. The first trivial idea is to compute the Pareto set at fixed points of time, e.g. every minute (assuming that the dimension of the problem is low enough). This computation can be performed in parallel to the computation of the path of solutions, so that the assignment of the optimal solutions never has to be interrupted. The computation of the entire Pareto set can also be started anew in reaction to external influences. As an example in case of the linear motor some precomputed velocity profiles are contained in the optimization. If this profile cannot be realized any more, e.g. because the shuttle has to brake unexpectedly, a recomputation of the entire Pareto set and the corresponding path (including the new profiles) is necessary.

5 Application and Results

5.1 The Railcab-System

The selected validation base for the self-optimization scheme within project SFB 614 [9] is a novel linear-motor driven railway system, developed by a project called "Neue Bahntechnik Paderborn" [10]. Figure 3 displays the test vehicle developed within this project. This vehicle belongs to a test bed with a track length of about 530 m. The track contains an artificial hill with an altitude of about 2.5 m and gradients are up to 6%, requiring a varying thrust along the track. The track is completed by one novel passive switch, which allows a rapid termination of a compact convoy.

The vehicle consists of a superstructure that carries the load and two undercarriages. Figure 3 (right picture) shows the concept of the undercarriage module, which is one of the basic modules of the vehicle. The undercarriage houses three sub-modules: A driving and breaking-, an active suspension- and a guidance module, the latter based on one single wheel set. The doubly fed linear drive module serves three functions: The energy transfer from the primary to the secondary, the control of the pitch angle and the generation of thrust described in [15]. The active suspension module represents the means to realize improved comfort for passengers [6]. The guidance module [5] enables a driving with low attrition and the novel concept for a passive switch.

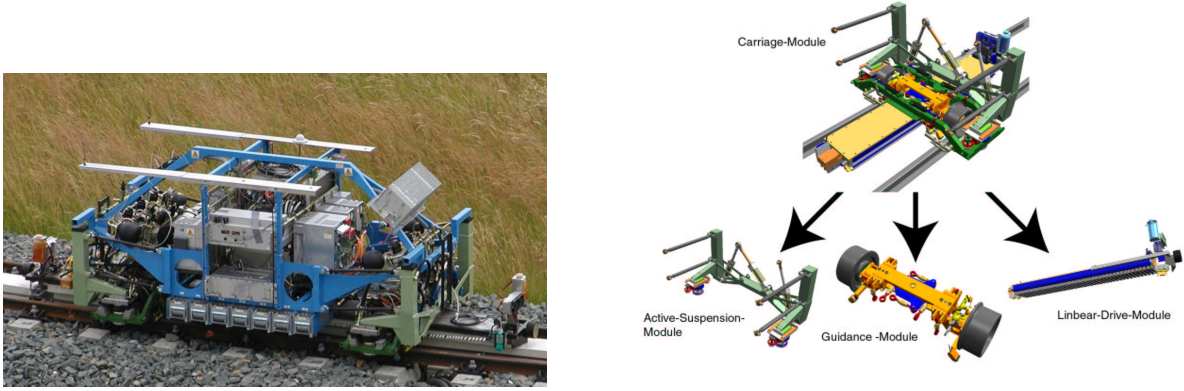


Figure 3: Vehicle of the demonstrator (left) and concept of the drive module (right)

5.2 Optimized Operating Point Assignment of a Linear Motor

The doubly fed linear drive

The doubly fed linear drive module comprises of a primary motor part between the rails of the track and a secondary part on the vehicle. Both parts have a power supply controlling the currents by power converters. The generated thrust depends on the product of the amplitude of the currents in the primary and the secondary. The direction of the thrust is the result of the phase difference between both currents. The phase difference depends on an electrical angle and the mechanical position of the secondary part. The current in the primary has to be determined for both driving and braking modules of one vehicle and in case of a second vehicle on the same track section for them too. Figure 4 illustrates the principle of the drive and the control structure of the doubly fed linear drive. The key function of this structure is the operating point assignment on the vehicle, which is focussed in this article. In this work, it is assumed that the other functional blocks in the shuttle like current-, speed- and position-control, profile generation and energy management do not have repercussions on the operating point assignment.

The operating point assignment is the responsible function regarding the controlled thrust generation, the power transfer from primary to the secondary motor part and conditions of the vehicle like speed and the actual stored energy in the vehicle batteries etc. An operating point of the doubly fed linear drive is characterized by the distribution of thrust forming currents and the frequency of current at the primary. This frequency and mechanical speed of the vehicle yields the frequency of current at the secondary.

The optimization task for the operating point assignment

The optimization task is to distinguish between a proper operating point with a special look at the efficiency of the drive and at the converter utilization factor. The efficiency considers the mechanical power resulting from thrust, speed and the electrical power transferred to the vehicle. For thrust generation ($F_M > 0$) the efficiency η_{LM} and the

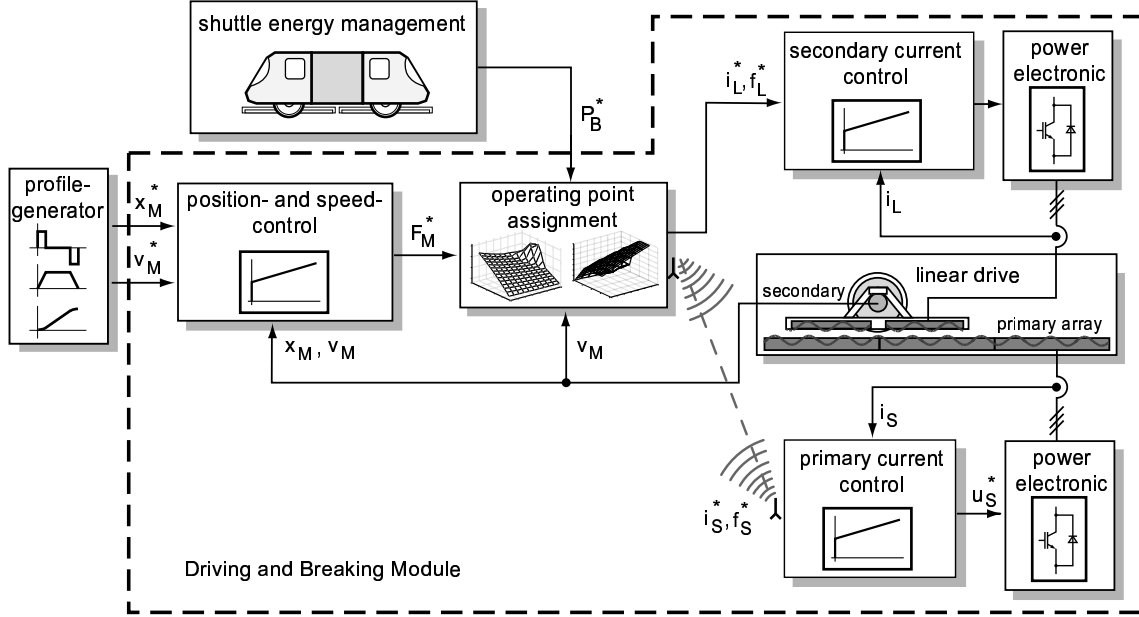


Figure 4: Control Structure of the Driving and Breaking Module

converter utilization factor η_{SN} are given by

$$\eta_{LM} = \frac{P_M + P_B}{P_S} \quad \text{and} \quad \eta_{SN} = \frac{P_M + P_B}{S_S + S_L}.$$

In case of braking ($F_M \leq 0$), the equations change to:

$$\eta_{LM} = \frac{P_B}{P_S + P_M} \quad \text{and} \quad \eta_{SN} = \frac{P_B}{S_S + S_L}.$$

P_M denotes the mechanical power of the drive, P_B the transferred power, S_S the apparent power of the primary motor part, S_L the apparent power of the secondary motor part. By definition an optimization of the efficiency leads to minimal required primary real power. The converter utilization factor represents the ratio of real output power to apparent power of the linear motor. The latter is a gage for the electrical copper losses in the secondary. The individual consideration of these two objectives leads to a conflicting solution. If the temperature in the secondary motor part rises and the cooling of this motor part is not sufficient, an η_{SN} -optimization should be preferred – otherwise an optimized η_{LM} should be used to determine the operating point. Hence, it is necessary to find an optimal compromise between these objectives. The thrust generation has the highest priority, the power transfer is also very important for the operation of the vehicle. Both goals are constraints for these optimizations. Results of a multiobjective optimization are presented in Figure 5 and [11].

Making use of the multiobjective optimization algorithm described in [2] this leads to a set of solutions for possible operating points (the Pareto set). Thus it is essential to select one of the solutions as result (decision heuristic). This choice is determined by the temperature of the secondary motor part and the charging state of the battery. A

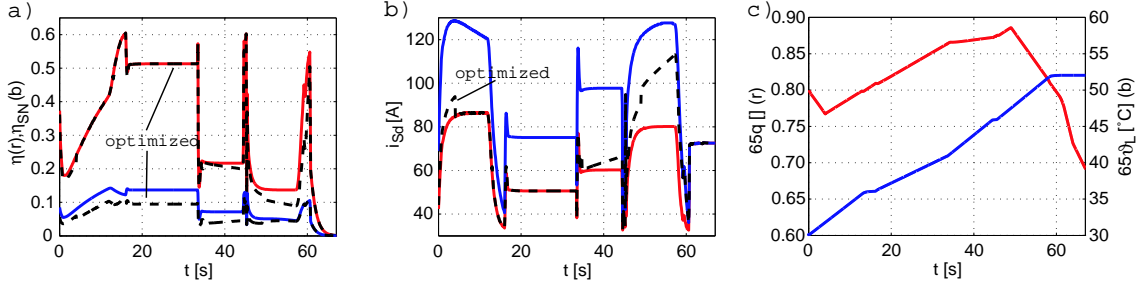


Figure 5: a): max. possible efficiency (red), max. possible converter utilization factor (blue) and values of multiobjective optimization (black) b): primary current references: degree of efficiency (red), converter utilization factor (blue), multiobjective optimization (black) c): temperature of secondary (red) and railcab battery load value (blue)

high temperature of the secondary part produces higher copper losses and low charging state a high converter utilization factor. A varying constraint e.g. required thrust or the energy transfer requires a new operating point. If the constraints show a known time depending value, an optimization can be computed and stored solutions are available. But in case of an unexpected change of this requirements or changing parameter in the linear drive, e.g. the resistance of the secondary or the motor induction, a new optimization has to be performed. Unfortunately it turned out that this computation needs too much time to compute it in real time. In this case a new strategy for tracking selected operating points influenced by changing constraints or parameters is required. This has been done by the presented numerical path following approach (cf. Section 4). The main advantage of this very fast algorithm is to prevent the time consuming calculation of a new Pareto set. In order to illustrate the quality of this method a comparison of the optimized and tracked operating points is necessary.

Results of tracking a path of an operating point

To generate an exemplary result for combination of the Pareto optimization and the path following strategy, the operation point assignment is calculated for a special maneuver. This maneuver is illustrated in Figure 6, which shows the position and velocity of the vehicle. The maneuver consists of acceleration and a braking term. While the vehicle accelerates the thrust shown in Figure 7 is positive.

In the braking term the thrust becomes negative. While it is small, the real transferred power can not be complied to constrain of requirements. In fact the temperature of the secondary motor part and the stored energy of the batteries depend on the chosen operating point. For this simulative-based optimization and path following application it is assumed that both values can be measured. To simplify the used model and to make the results comparable, the model considers pre-calculated values (see Figure 8) for the former measurements. Figure 9 illustrates the calculated Pareto sets and the chosen Pareto points in the objective space of efficiency and converter utilization factor at different points of time. Optimal points in the physical sense would be located in

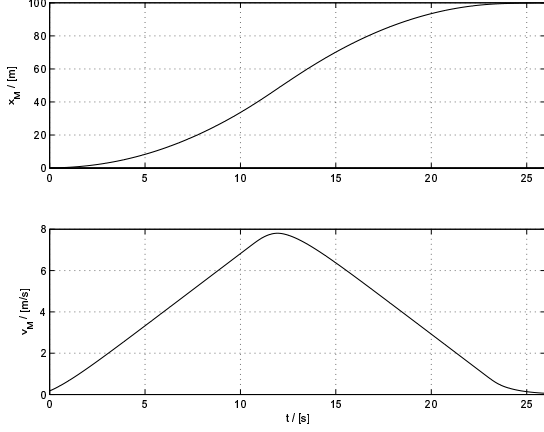


Figure 6: Profiles of position (above) and velocity (below)

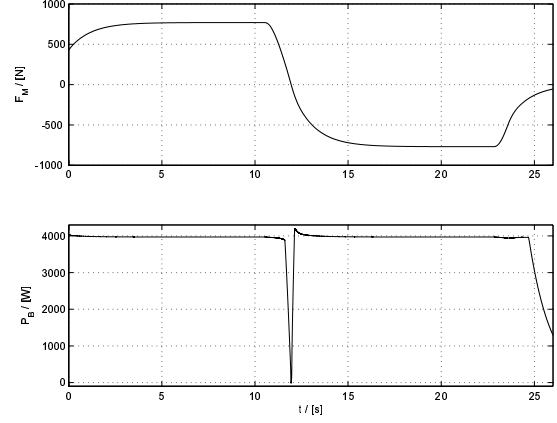


Figure 7: Profiles of thrust (above) and resulting transferred power (below)

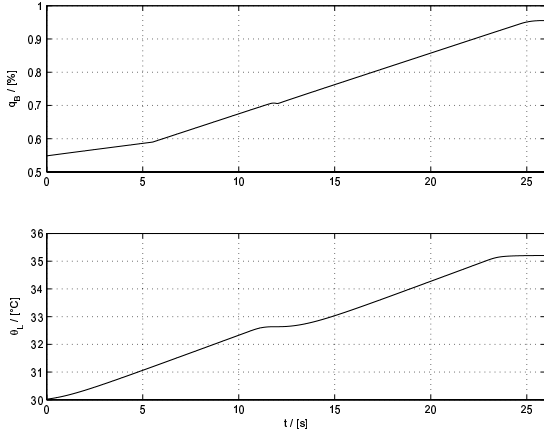


Figure 8: Stored energy rate (above), temperature of sec. motor part (below)

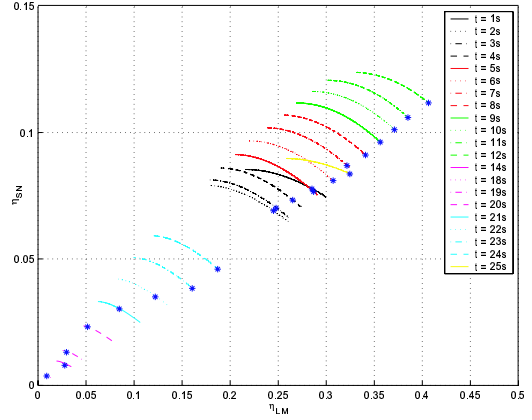


Figure 9: Pareto-optimal values in objective space

the upper right corner of the figure (i.e. high values for η and η_{SN}). For an increasing time – or an increasing thrust respectively, the Pareto sets approach this target.

Figure 10 shows some Pareto sets in preimage space – that is intervals of Pareto-optimal values for the current I_S – (red lines), the Pareto points which would have been selected by an adequate decision heuristic (blue stars) and the computed path (black). As described in [11] the decision heuristic is based on the measurements of the temperature and the stored energy rate. For higher temperatures or a lower value of the stored energy rate respectively, Pareto points with a relatively high converter utilization factor are preferred.

Having used this decision heuristic with a specified weighting of the temperature and the stored energy rate (cp. Figure 9), the computed optimal adjustments are located on the lower boundary of the Pareto sets. The quality of the combination of the multiobjective optimization and path following techniques is proven in Figure 10 by the low distance

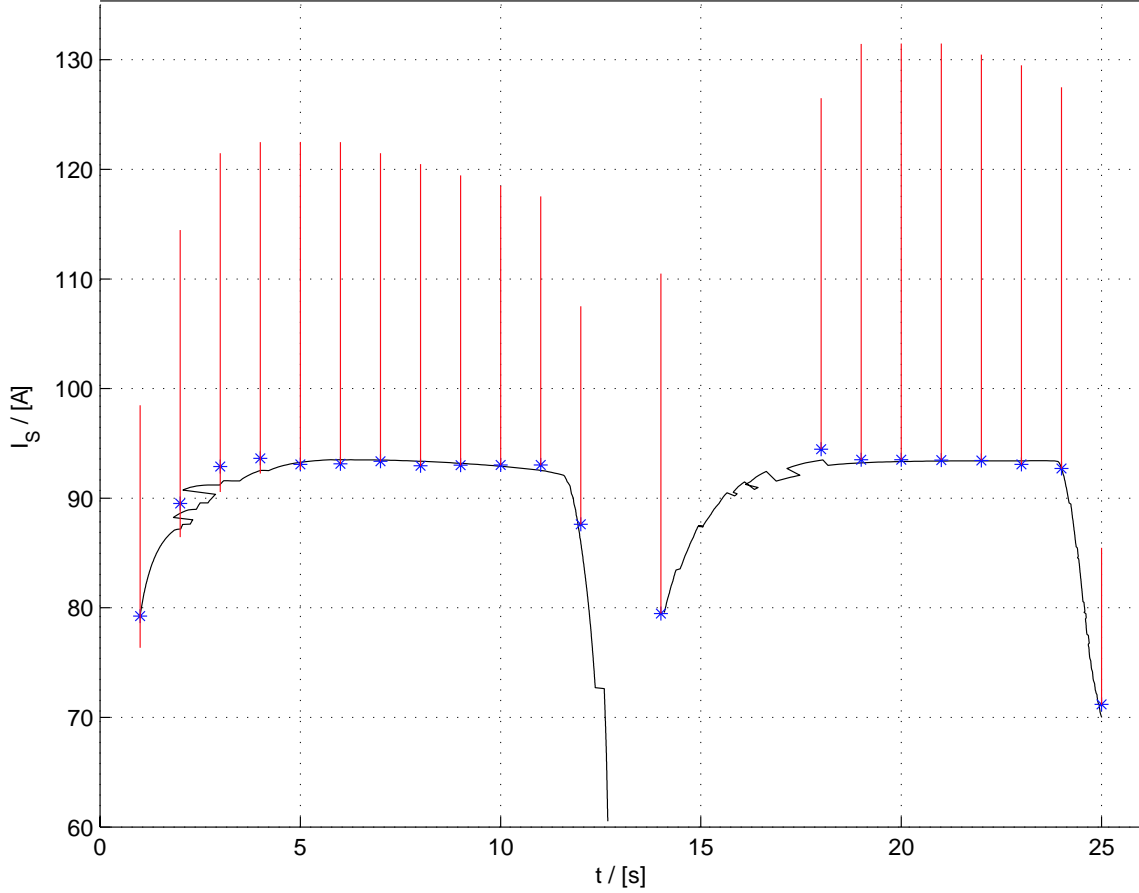


Figure 10: Comparison of by the decision heuristic selected Pareto points and the computed path in preimage space (t versus I_S)

between the followed path and the chosen Pareto points. In case of the low thrust requirements the transferred power is conflicting to the constrained power transfer. This leads to a disruption of the followed path; the following has to be restarted with an new Pareto point.

6 Conclusion

In general a Pareto set of a time-dependent problem is based on old information at the time when it is completely computed. The problems resulting from this insufficiency increase with longer simulation times. Therefore a global Pareto optimization approach is unpracticable for online applications. A possible way out of this dilemma is the combination of multiobjective optimization and numerical path following presented in this work. Having computed one Pareto set (e.g. for $t = 0$) and having applied to this set a decision heuristic (which takes information about the function values into account), the path can be computed until the new Pareto set is available. The selected Pareto point and the path that has been followed can be adjusted online. In case of

the optimization studied in this work it took about 3 seconds to compute an adequate covering of the entire Pareto set and about 0.5 seconds to compute a path which provides us with solutions for a time interval of 11 seconds. The remaining time (in this case 10.5 seconds) is long enough for the computation of a new Pareto set, e.g. the entire one for $t = 11$ s, assuming known profiles for the maneuver.

Although we can only guarantee that the computed points in the path are substationary points, the results for the operating point assignment as described above lie within the Pareto-optimal set. Even if one leaves the Pareto set to some substationary points which are not Pareto points, restarting the path on new entire Pareto sets will correct this problem.

Acknowledgement

This work was partly developed in the course of the Collaborative Research Center 614 – Self-Optimizing Concepts and Structures in Mechanical Engineering – University of Paderborn, and was published on its behalf and funded by the Deutsche Forschungsgemeinschaft.

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