THE IMAGE OF THE HEAT TRANSFORM ON SYMMETRIC SPACES

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The heat equation on $\mathbb{R}^n$ is the partial differential equation

$$\begin{aligned}\Delta u(x,t) &= \frac{\partial}{\partial t} u(x,t)\\ \lim_{t\to 0^+} u(x,t) &= f(x)\end{aligned}$$

where f is a L^2-function on $\mathbb{R}^n$ and $\Delta = \sum \partial^2/\partial x_i^2$ is the Laplace operator. The solution is given by

$$u(x,t) = H_t f(x) = e^{t\Delta} f(x) = \frac{1}{(4\pi t)^{n/2}} \int_{\mathbb{R}^n} f(y) h_t(x-y) = \int_{\mathbb{R}^n} f(y) e^{-(x-y)^2/4t}\, dy\,.$$

The transform $H_t : f \mapsto H_t f$ is the *Heat transform.* It can be read of from this formula, that $H_t f$ has a holomorphic extension to all of $\mathbb{C}^n$. It is a classical result, that the image of the Heat transform is the space of holomorphic functions $F : \mathbb{C}^n \to \mathbb{C}$, such that

$$\|F\|_t^2 := (2\pi t)^{-n/2} \int |F(x+iy)|^2 e^{-\|y\|^2/2t}\, dxdy < \infty$$

and $\|f\| = \|H_t f\|$.

The Heat equation has a natural generalization to all Riemannian manifolds. The solution is again given by the Heat transform

$$u(x,t) = H_t f(x) = \int f(y) h_t(y)\, dy$$

where h_t is the *heat kernel*, but there is no "natural" complexification in general, and hence it is not clear how to realize the image in a space of holomorphic functions.

In this talk, we will discuss the Heat transform on $\mathbb{R}^n$ in some details to modify the concepts and ideas that are needed for Riemannian symmetric spaces of the form G/K where G is a connected semisimple Lie group and K a maximal compact subgroup. The second part is more abstract and deals with the results in a joint work with B. Krötz and R. Stanton on the image of the Heat transform on G/K. The main tools here are the spherical Fourier transform and the Abel transform.

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