ALL TILTING MODULES ARE OF COUNTABLE TYPE

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Let R be a ring. A (right R-)module T is called tilting, if:

(1) T has finite projective dimension,
(2) $\mathrm{Ext}^i_R(T, T^{(\kappa)}) = 0$ for all $1 \leq i < \omega$ and all cardinals κ, and
(3) there is an exact sequence $0 \to R \to T_0 \to \cdots \to T_m \to 0$ such that T_i is a direct summand in a direct sum of copies of T for each $i \leq m$.

In the joint work with Jan Trlifaj we show that T is then of countable type. That is, there is a set, $\mathcal{C}$, of modules possessing a projective resolution consisting of countably generated projective modules such that the tilting class $T^\perp$ equals $\mathcal{C}^\perp$.

Moreover, a cotorsion pair $\mathfrak{C} = (\mathcal{A}, \mathcal{B})$ is cogenerated by a tilting module if and only if $\mathfrak{C}$ is hereditary, $\mathcal{B}$ is closed under arbitrary direct sums, and there is an $n < \omega$ such that all modules in $\mathcal{A}$ have projective dimension $\leq n$.

References

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