An Optimization-Based Empirical Mode Decomposition (OEMD)

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Traditional Empirical Mode Decomposition (EMD)

Huang et al. [1]: Empirical Mode Decomposition (EMD) decomposes signal $f$ additively into intrinsic mode functions (IMFs) $f_k(t)$

$$f(t) = \sum_{k=1}^{n} a_k(t) \cos(\omega_k(t) \cdot r_{s_k}(t))$$

For $k$-th IMF: $a_k(t) \geq 0$ is envelope

Definition: $a_k(t) \geq a_{k+1}(t)$ for all $t$

Equality holds at extrema $f(t^*)$ or $f(t^-)$

Generation of envelope by cubic spline interpolation of extrema

⇒ over/under-shooting and destruction of some physical properties of IMFs

Hou et al. [2]: EMD

$$f(t) = \sum_{k=1}^{n} a_k(t) \cos(\omega_k(t) \cdot r_{s_k}(t))$$

Optimization-Based Empirical Mode Decomposition

New Optimization-Based Empirical Mode Decomposition (OEMD) defines envelope for (1) strictly mathematically through optimization process:

(P1) Minimize $\| S(X(t)) \|_1$ over all $X(t)$

Subject to $f(t) \leq X(t)$, and $f(t) = X(t)$

(P2) Minimize $\| S(X(t)) \|_1$ over all $X(t)$

Subject to $f(t) \geq X(t)$, and $f(t) = X(t)$

Here: $f(t)$ given function in $k$-th sifting process $t^* \leq t \leq t^-$

Position of maxima, minima $f(t^+), f(t^-)$

Local maxima, minima $\delta_t$

Smoothness functional (e.g., n-th derivative, n-th total variation)

Solutions: $a_k(t), a_{k+1}(t)$ upper, lower envelope

(See [2] for a different optimization approach)

Solution of optimization problems (3) by CVX tool box [3,4].

1-D OEMD

Smoothness functional $\delta_t$: 3rd order total variation

Test signal from [5] consisting of three components: a sinusoid of some medium period $T$ superimposed by two triangular waveforms with periods smaller than larger than $T$.

Results of decomposition:

Original signal from [5]

IMFs 1 to 3 from left to right: Original component, IMFs from [5] and IMFs from new OEMD

Difference of original components and IMFs: difference between original components and [5], difference between original components and OEMD

Instantaneous Frequency Analyses

Improvement of normalization based instantaneous frequency analyses from [6]:

Step 1: solve optimization problem

(P3) Minimize $\| S(X(t)) \|_1$ over all $X(t)$

Subject to $f(t) \leq X(t)$, and $f(t) = X(t)$

Solution: $a(t)$ is envelope of $f(t)$

Step 2: define

$$\cos(\omega(t)) = \frac{a(t)}{f(t)}$$

Example from [7]: signal $f(t) = \sin(8\pi \sin(2\pi t) + t)$, $t \in [0,1]$

Sampled on non-uniform grid

with sampling frequency 1024 Hz in [0.0, 0.5], and 512 Hz in [0.5, 1]

Signal sampled on non-uniform grid

IMF in [7]

Optimization-Based Empirical Mode Decomposition

Naive 2-D OEMD:

compute extrema in each direction independently in tensor product approach

(see compare to 2-D wavelet decomposition in JPEG-2000)

Test image: A portion of D102 in the Brodatz image set [8]

Smoothness functional $\delta_t$ is 3rd order variation

Test image:

- (a) Test image
- (b) IMF1
- (c) IMF2
- (d) Residual image

Observation: many noise-like stripes in every IMF image

Reason:

method does not consider local extrema of image in both directions simultaneously

Bivariate OEMD:

Step 1: find extrema of image by comparing pixel value with all neighbors in $k \times k$ subimage

Step 2: solve optimization problems (P1) and (P2) for $X(i,j)$

Smoothness functional $\delta_t$ is first order 2-D total variation

Test image 1: A portion of D102 in the Brodatz image set [8]

Test image 2: natural photo “Baboon”

Test image 3: natural photo “Babara”

References:


